Test/
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Math. 30
Aug. 1969

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HIGH SCHOOL AND UNIVERSITY MATRICULATION
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DEPARTMENTAL EXAMINATIONS, 1969
AUGUST

MATHEMATICS 30

Time — 2 hours.

DO NOT SPEND TOO MUCH TIME ON ANY ONE QUESTION.
READ THE PAPER CAREFULLY.

Total Possible Mark — 104.

NOTE — This examination (Questions 1 - 52) is completely machine scored. Knott's Mathematical Tables may be used and these tables will be supplied by the Presiding Examiner. Candidates are NOT permitted to use slide rules.

Your score on this test will be determined by the number of correct answers.
There will be no deduction for wrong answers.

> To ensure accuracy in the handling of your examination papers, the following sections on your ANSWER SHEET MUST BE COMPLETED.
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> SECTION A — Supply information as requested.
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O NOT FOLD EITHER THE ANSWER SHEET OR THE TEST BOOKLET.

our presiding examiner will collect the answer sheet and booklet separately for transmission to e Department.

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Questions 1 - 52 inclusive, have a value of two marks each. Answers to these questions are to be recorded on the separate ANSWER SHEET which will be supplied by the Presiding Examiner. Each question has four suggested answers of which only one is correct. In each case select the CORRECT ANSWER and record your choice on the separate ANSWER SHEET as shown in the example below. There is no penalty for incorrect answers.

EXAMPLE

The product of 8 and 12 is

ANSWERS

A. 20
B. 96
C. 48
D. 72

ANSWER SHEET

A	B	C	D	E
☐	■	☐	☐	☐

DISREGARD COLUMN "E"
ON ANSWER SHEET

You are to use an ORDINARY HB pencil to mark your answers on the answer sheet. Make certain your answer marks are heavy and black. If you change your mind about an answer, be sure to erase your first mark completely. There should be only one answer marked for each question. Be sure there are no stray pencil marks on your answer sheet.

1. The truth value of the statement $P \rightarrow Q$ can best be described by which entry in the table?

	P	Q	P → Q
A.	T	T	F
B.	T	F	T
C.	F	T	T
D.	F	F	F

2. If $\oplus$ is addition modulo 6, then $4 \oplus 4$ is

A. 0
B. 1
C. 3
D. 2

3. Which of the following statements is true?

A. The set of negative even integers is closed under addition.
B. The set of negative integers is closed under multiplication.
C. The set of odd integers is closed under addition.
D. The set of negative even integers is closed under multiplication.

4. Under which one of the conditions below is the equation $(\sqrt{a})(\sqrt{b}) = \sqrt{ab}$ true?

A. $\forall a,\ \forall b,\ (\sqrt{a})(\sqrt{b}) = \sqrt{ab},\ a, b \in Q$
B. $\forall a,\ \forall b,\ (\sqrt{a})(\sqrt{b}) = \sqrt{ab},\ a, b \in N$
C. $\forall a,\ \forall b,\ (\sqrt{a})(\sqrt{b}) = \sqrt{ab},\ a, b \in R$
D. $\forall a,\ \forall b,\ (\sqrt{a})(\sqrt{b}) = \sqrt{ab},\ a, b \in I$

FOR ROUGH WORK
(No marks given for work in this space)

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[OVER]

5. How many elements has the solution set of the equation $(x + 4)(x - 2) = (x + 5)(x - 3)$?

A. none
B. one
C. two
D. an infinite number

6. If $f : x \rightarrow 3x + 7,\ x \in R$, then f^{-1} may be expressed as:

A. $f^{-1} : x \rightarrow \frac{x}{3} - 2\frac{1}{3},\ x \in R$

B. $f^{-1} : x \rightarrow 3x - 7,\ x \in R$

C. $f^{-1} : x \rightarrow \frac{x + 7}{3},\ x \in R$

D. $f^{-1} : x \rightarrow 3(x - 7),\ x \in R$

7. Given the arithmetic sequence 6, 11, 16, . . . , which term in the sequence is 426?

A. 83rd term
B. 85th term
19 C. 84th term
D. 71st term

8. If a, b, c, d, are in geometric sequence, then which of the following relations is false?

A. $bc = ad$
14 B. $ac = bd$
C. $b^2 = ac$
D. $c^2 = bd$

9. How many terms in the series $35 + 30 + 25 + \ldots$ are required to yield a sum of -40?

A. 14
14 B. 15
C. 16
D. 18

10. Given $P = 2x^2 - 3x + 4$, then the minimum value of P is

A. $\frac{3}{4}$

B. $1\frac{1}{2}$

C. $2\frac{7}{8}$

D. $3\frac{7}{16}$

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FOR ROUGH WORK
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11. For the equation $\sqrt{x} - 1 = \sqrt{2x} + 1$, $x \in R$, the solution set is

A. $\{0\}$
B. $\{4\}$
C. $\{0, 4\}$
D. $\{\emptyset\}$

12. The present value of a \$500 annuity payable every other year for 20 years, interest compounded at 6% annually is

A. $\dfrac{500(1.06^{20} - 1)}{1.06^2 - 1}$

B. $\dfrac{500(1 - 1.06^{-10})}{1.06^2 - 1}$

C. $\dfrac{500(1 - 1.06^{-20})}{1 - 1.06^2}$

D. $\dfrac{500(1 - 1.06^{-20})}{1.06^2 - 1}$

13. The total cost of a banquet is a linear function of the number of guests. When 160 guests attend, the cost is \$1,032; and when 225 attend, the cost is \$1,422. The cost when 240 guests attend would be

A. \$1,548
B. \$1,512
C. \$1,516.80
D. \$1,532

14. In proving $3^n - 1$ divisible by 8 for all even natural numbers, which of the following steps in the mathematical induction proof is in error?

Step 1. For $n = 2$, $f(2) = 3^2 - 1 = 8$, which is divisible by 8.

Step 2. Assume for $n = 2k$, that $f(2k) = 3^{2k} - 1$ is divisible by 8.

Step 3. For $n = 2k + 1$,
$$\begin{aligned} f(2k+1) &= 3^{2k+2} - 1 \\ &= 3^{2k+2} - 3^2 + 3^2 - 1 \\ &= 3^2(3^{2k} - 1) + 8 \end{aligned}$$

Hence if $f(k)$ is divisible by 8, so also is $f(k + 1)$.

Step 4. Therefore $3^n - 1$ is divisible by 8 for all even natural numbers.

A. Step 1
B. Step 2
C. Step 3
D. Step 4

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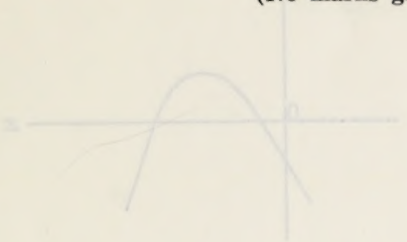
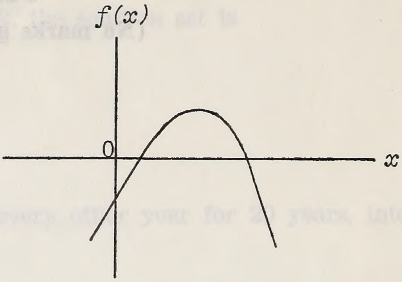


15. If the graph of the quadratic function f : $x \rightarrow ax^2 + bx + c$ is depicted as in the sketch, then analysis proves which conditions must exist?

A. $c < 0; \quad a > 0$
B. $a < 0; \quad b < 0$
C. $b^2 - 4ac > 0; \quad b < 0$
D. $b > 0; \quad a < 0$



16. If the function $x^2 - 4x + c$ has a minimum value of 20, then the value of c is

A. 9
B. 84
C. 24
D. 2

17. The domain for which the function $10 - 13x - 3x^2$ is positive is

A. $\{x \mid -\frac{3}{2} < x < \frac{1}{5}, x \in R\}$
B. $\{x \mid -5 < x < \frac{2}{3}, x \in R\}$
C. $\{x \mid -\frac{1}{5} \leqq x \leqq \frac{3}{2}, x \in R\}$
D. $\{x \mid -5 \leqq x \leqq \frac{2}{3}, x \in R\}$

18. The statement which most completely describes the roots of the equation $2x^2 + x - 15 = 0$ is

A. The roots are real and equal.
B. The roots are real and unequal.
C. The roots are real, distinct, and rational.
D. The roots are real, distinct, and irrational.

19. If the sum of the first 8 terms of an arithmetic series is 52, and the sum of the first 16 terms is –88, then the fifth term is

A. 8
B. 11 (14)
C. 5
D. 2

20. If the 7th term of a geometric sequence is –1,458, then

A. $a_1 < 0$
B. $a_1 > 0, r = -3$ (14)
C. $a_1 > 0, r < 0$
D. $a_1 > 0$

FOR ROUGH WORK
(No marks given for work in this space)

21. The reciprocal of the conjugate of the complex number –9 is

A. $\frac{i}{9}$

B. $\frac{1}{9}$

C. $-\frac{i}{9}$

D. $-\frac{1}{9}$

22. The graph of $y^2 = 16 + x^2$ is

A. an hyperbola
B. an ellipse
Geo
C. a parabola
D. a circle

23. The equation of the circle with center (5, –1) and on the point (3, 1) is

A. $(x + 5)^2 + (y - 1)^2 = 8$
B. $(x - 5)^2 + (y + 1)^2 = 8$
Geo
C. $(x + 5)^2 + (y - 1)^2 = 2\sqrt{2}$
D. $(x - 5)^2 + (y + 1)^2 = 2\sqrt{2}$

24. The conjugate of the product of $(\sqrt{2} + 7i)(-\sqrt{2} - 3i)$ is

A. $19 - 10i\sqrt{2}$
B. $2 + 21i\sqrt{2}$
C. $19 + 10i\sqrt{2}$
D. $-2 - 21i\sqrt{2}$

25. The graph of the relation $8 + y^2 = 4x^2$ has symmetry with respect to

A. both coordinate axes only
B. both coordinate axes and the origin
Geo
C. the origin only
D. none of these

26. The radius of the circle whose equation is $2x^2 + 2y^2 - 16x + 5y + 10 = 0$ is

A. $\frac{1}{4}\sqrt{351}$
B. $\frac{1}{4}\sqrt{101}$
Geo
C. $\frac{1}{2}\sqrt{201}$
D. $\frac{1}{4}\sqrt{201}$

27. $2i^3 + 3i^4 + 5i^5$ is the equivalent of

A. $3 + 7i$
B. $3 - 7i$
C. $-3 + 3i$
D. $3 + 3i$

FOR ROUGH WORK

(No marks given for work in this space)

28. The equation with roots $\frac{1}{2} \pm \frac{1}{2}i\sqrt{2}$ is

A. $4x^2 + 4x + 3 = 0$
B. $x^2 - x + 3 = 0$
C. $4x^2 - 4x = 3$
D. $4x^2 - 4x + 3 = 0$

29. The sum of the absolute values of $5 - 12i$ and $-3 + 4i$ is

A. -6
B. 18
C. 24
D. $\sqrt{194}$

30. The additive inverse of $-(-5 + 12i)$ is

A. $(-12 + 5i)$
B. $-(-12i - 5)$
C. $-(-12i + 5)$
D. $(5 + 12i)$

31. The y-intercepts of the relation $4x^2 = 9y^2 - 1$ are

A. $\frac{1}{2}, -\frac{1}{2}$
B. $3, \ -3$
C. $\frac{1}{3}, -\frac{1}{3}$
D. none of these

32. The domain of the relation $P = \{(x, y) \mid y^2 = -4x,\ x, y \in R\}$ is

A. $\{x \mid x \leqq 0\}$
B. $\{x \mid x \leqq 4\}$
C. $\{y \mid y \geqq 0\}$
D. $\{y \mid y \leqq 0\}$

33. One focus of the graph of the relation $4y^2 + 9x^2 = 36$ is

A. $(0, -\sqrt{5})$
B. $(\sqrt{5}, 0)$
C. $(0, 5)$
D. $(-5, 0)$

34. In the equation $(x - yi)(2 + i) = 1$ the values of x and y are

A. $x = \frac{2}{5}, y = \frac{1}{5}$
B. $x = \frac{1}{5}, y = \frac{2}{5}$
C. $x = \frac{2}{3}, y = \frac{1}{3}$
D. $x = \frac{1}{3}, y = \frac{2}{3}$

FOR ROUGH WORK

(No marks given for work in this space)

35. The range of the relation $Q = \{(x, y) \mid 49y^2 = -9x^2 + 441,\ x, y \in R\}$

A. $\{y \mid -7 \leqq y \leqq 7\}$
B. $\{y \mid -3 \leqq y \leqq 3\}$
C. $\{x \mid -7 \leqq x \leqq 7\}$
D. $\{x \mid -3 \leqq x \leqq 3\}$

36. Set $S = \{3, 4, 7, 8, 9\}$. The number of possible 3-arrangements, without repetition, is

A. $\binom{5}{3}$
B. $5! \times 3!$
C. $5! \times 3$
D. $(5)_3$

37. Using 5 letters from the word EQUATION, how many different "words" could be formed that begin with E and end with T?

[✓ A.] 120
B. 840
C. 720
D. none of these

38. One card is drawn at random from an ordinary deck of 52 playing cards. What is the probability that it is an ace or a diamond?

A. $\frac{1}{52}$
B. $\frac{17}{52}$
C. $\frac{1}{13}$
D. $\frac{4}{13}$

39. How many teams of four can be selected from seven prospects in a club, if the president of the club has to be included on each team and he is listed among the prospects?

A. $\binom{7}{4}$
B. $(7)_4$
C. $\binom{7}{3}$
D. $\binom{6}{3}$

40. If $P(A) = 0.7$, and $P(B) = 0.4$, which of the following would have to be false?

A. $P(A \cap B) = 0.2$
B. $P(A \cap B) = 0.8$
C. A and B are NOT mutually exclusive events
D. $P(A \cap B) = 0.28$

FOR ROUGH WORK

(No marks given for work in this space)

41. There are 3 red chickens, 2 black chickens, and 25 white chickens in a flock. A farmer goes into his chicken roost at night for a random selection of one chicken. The probability of selecting a red chicken or a black chicken is

A. $\frac{1}{6}$

B. $\frac{1}{12}$

C. $\frac{1}{5}$

D. $\frac{6}{25}$

$\frac{5}{30} = \frac{1}{6}$

42. The fifth term in the expansion of $\left(\frac{1}{x} - \sqrt{x}\right)^{10}$ is

A. $-\binom{10}{4}x^{-4}$

B. $-\binom{10}{6}x^{8}$

C. $\binom{10}{6}x^{-1}$

D. $\binom{10}{4}x^{-4}$

43. The term which is independent of x in the expansion of $\left(x^2 - \frac{1}{x}\right)^{12}$ is the

A. 9th term
B. 13th term
C. 8th term
D. 5th term

44. $\frac{(n-1)!}{(n-4)!\,3!} + \frac{(n-1)!}{(n-3)!\,2!}$ is equal to

A. $n(n-1)$

B. $n(n-1)(n-2)$

C. $\frac{n(n-1)(n-2)}{6}$

D. $\frac{n!}{6}$

45. There are 5 men and 6 women to choose from for a committee meeting. The number of different ways you can select a committee of 3 is

A. $(5)_3 \times (6)_3$

B. $3!$

C. $\binom{11}{3}$

D. $(11)_3$

FOR ROUGH WORK

(No marks given for work in this space)

46. A railway coach has ten seats facing backwards and ten facing forwards. In how many ways can eight passengers be seated if two refuse to face forward and three refuse to face backwards?

A. $(10)_2 \times (10)_3 \times (15)_3$

B. $\binom{10}{2}\binom{10}{3}$

C. $(10)_2 + (10)_3 + (15)_3$

D. $(10)_2 \times (15)_3$

8p

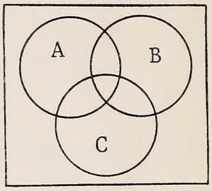
47. From the Venn diagram on the right, an expression equivalent to $(B - A) - C$ is

A. $(B \cup A) - C$

B. $A' \cap C' \cap B$

C. $A' \cup C'$

D. $(A - B) \cap (B - C)$



48. The general term for the expansion of $(4 + 3x)^8$ is

A. $\binom{8}{r}4 \cdot x^{8-r}3x^r$

B. $\binom{8}{r}(4)^{8-r} \cdot 3x^r$

15

C. $3^r\binom{8}{r}4^{8-r}x^r$

D. $\binom{8}{r}12^{8-r}x^r$

49. A gaming device has the form of a pyramid with four triangular faces. Each face is congruent, and each face has equal probability of facing down on each throw. The faces are numbered 1, 2, 3, 4. The probability of having the 4 land face down exactly 3 times in four throws is

A. $\binom{4}{3}\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^3$

16

B. $\binom{4}{3}\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)$

C. $\binom{4}{3}\binom{4}{2}\binom{4}{1}$

D. $\binom{4}{3}+\binom{4}{2}+\binom{4}{1}$

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Departmental examinations,
[grade 12] 1969, August. -

39780803 CURR HIST

50. An urn contains three black balls, two white balls, and two red balls. Two balls are drawn at random. If the first ball is replaced before the second is drawn, the probability that both balls drawn will be black is

A. $\frac{6}{14}$

B. $\frac{9}{49}$

⑭ C. $\frac{6}{7}$

D. $\frac{6}{49}$

51. If $A = \{z \mid |z + 4 - 2i| > 4,\ z \in C\}$, then the graph of A is

A. the interior of a circle
B. a circle
C. a circle plus its exterior
D. the exterior of a circle

52. The probability that a certain man will be alive 25 years hence is $\frac{3}{7}$, and the probability that his wife will be alive 25 years hence is $\frac{4}{5}$. The probability that, 25 years hence, only the man will be alive is

A. $\frac{3}{7}$

⑯ B. $\frac{12}{35}$

C. $\frac{3}{35}$

D. $\frac{1}{5}$